Communications Eng. Department Level: 3rd Year Examiner: Dr. Mohamed Eid Time allowed: 3 hours	Semester: Autumn 2019 Final Exam: Mathematics IV Code: Math 301 Date: January 5, 2020

The Exam consists of one page — Answer all questions — No. of questions: 4 — Total Mark: 55

Question 1 (15 marks)

(a)Find the integrals: (i) $\int_0^\infty x^6\, e^{-2x}\, dx$ (ii) $\int_0^\infty \frac{1}{1+x^4}\, dx$ (iii) $\int_0^{\frac{\pi}{2}} \sqrt{\cot x}\, dx$ 6

(b)If $J_k(x)$ is the Bessel's function:

(i) Show that: $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x.$ 4

(ii)Find $\int_0^1 x^3\, J_0\, dx$ where $J_0(1) = 0.77,\ J_1(1) = 0.44.$ 5

Question 2 (10 marks)

(a)Solve the PDE: (i) $u_{xx} - u_{yy} = e^{2x+3y}$ (ii) $3u_{xy} - u_{yy} = \cos(3x-2y)$ 4

(b)Solve the wave equation: $u_{tt} - u_{xx} = 0,\quad 0 < x < 1$

$\text{B.C. } u(0,t) = u(1,t) = 0$ and $\text{I.C. } u(x,0) = x,\quad u_t(x,0) = 3.$ 6

Question 3 (15 marks)

(a)Write the table of frequency and the Pdf of the data: 2, 2, 2, 3, 3, 5, 5, 5, 6, 6, 8, 8. 3

Also, find $\bar{x},\ \sigma.$

(b)From the data: (1, 3), (2, 5), (4, 7), (5, 11), (7, 14).

Find the regression line $y = a + bx$ and $\bar{x},\ \bar{y},\ \sigma_x,\ \sigma_y$, $\text{cov}(x,y)$, r. 4

(c) If x is random variable with pdf $f(x) = \frac{1}{4}(x+1),\ 0 \le x \le 2$. Find the moment

generating function $M_x(t)$ and from it, find $m_1,\ m_2,\ and\ \sigma.$ 4

(d)If x, y are random variables with pdf $f(x,y) = \frac{1}{40}(x^2 y),\ x = 1, 2,\quad y = 0, 2, 3.$ 4

Find $\text{cov}(x,y)$.

Question 4 (15 marks)

(a)From Beta distribution, show that $\sigma = \sqrt{\frac{m.n}{(m+n+1)(m+n)^2}}.$ 3

(b)If the probability of a defective item in production processing is 0.002. By the binomial distribution, find the probability that a lot of 300 items contains 1 defective. 4

(c)If $\mu = 0.8$, $\sigma = 2$ in normal distribution. Find $P(2 \le x \le 3)$, $P(x > 3)$ where 4

$\phi(1.1) = 0.8643,\ \phi(0.6) = 0.7257$

(d)From the Gamma distribution: $f(x) = \frac{1}{\Gamma(n)} x^{n-1} e^{-x},\quad x, n > 0$ 4

Find $P(x \le 3)$ and $P(x > 4)$ when n = 2.

Good Luck — *Dr. Mohamed Eid*